PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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The U.S. Navy Doppler geodetic system and its observational accuracy

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This paper describes the configuration and general operations of the Doppler system Tranet that is supported by the U.S. Navy for geodetic research. It also discusses all sources of observational error that contribute more than 1 m in the calculation of position from Doppler data. The current observational accuracy of the system is estimated to be 10 m for the data from a single pass of a near Earth satellite.

1. Introduction: general description of the system

Beginning in 1959 and continuing to the present, the United States Navy has built up and maintained a system and a programme for geodetic research using measurements of the Doppler shift in radio receptions from artificial Earth satellites. The system involved is frequently called the Tranet system and the programme is frequently called the Anna programme. Technical responsibility for the ground system and for the satellites has been placed at the Applied Physics Laboratory of the Johns Hopkins University. In addition to the satellites that have been launched under the sponsorship of the Navy, three satellites in the geodetic programme have been sponsored by the N.A.S.A., namely the Beacon Explorers B and C and the Geos Explorer A. Geodetic analysis of data from this system has been carried out at the Applied Physics Laboratory and at the Naval Weapons Laboratory.

When the first satellite in the programme was launched in 1959 the system contained six stations. Four of these were in the continental U.S., one was in Newfoundland, and the sixth was in England. Three of the six are still operating as part of the present system of 14 stations. Figure 1 shows the approximate locations of the present stations, together with the geographical name and the identifying number associated with each station. The site marked APL/JHU includes the station indicated by the number 111. It also includes the telemetry and command station indicated by the number 502 and the communications centre indicated by the initials SCC.

The locations of the stations were not chosen at random. On an ideal globe the stations would presumably be spaced uniformly on the surface.† Since three-quarters of the Earth's surface is covered by water it is not convenient to have uniform distribution. In choosing sites on the actual Earth, we applied three main criteria: (*a*) feasibility of establishing and maintaining a station; (*b*) minimization of the maximum time interval between observations, for satellites at all orbital inclinations; and (*c*) occupancy for as long a time as possible.

† For a uniform distribution, the stations would have to be placed at the vertices of a regular polyhedron. Since there are only five regular polyhedra, an 'ideal' observation system would have to contain 4, 6, 8, 12, or 20 stations.

The reason for the first criterion is obvious. The others follow from the methods used in our research. We gain geodetic information by analysing perturbations in the positions of a number of satellites, where by perturbations we mean any deviations from a fixed elliptic orbit. Some perturbations are secular. Others are approximately periodic, with periods ranging from years or months down to an hour or so. Analysis of short-period

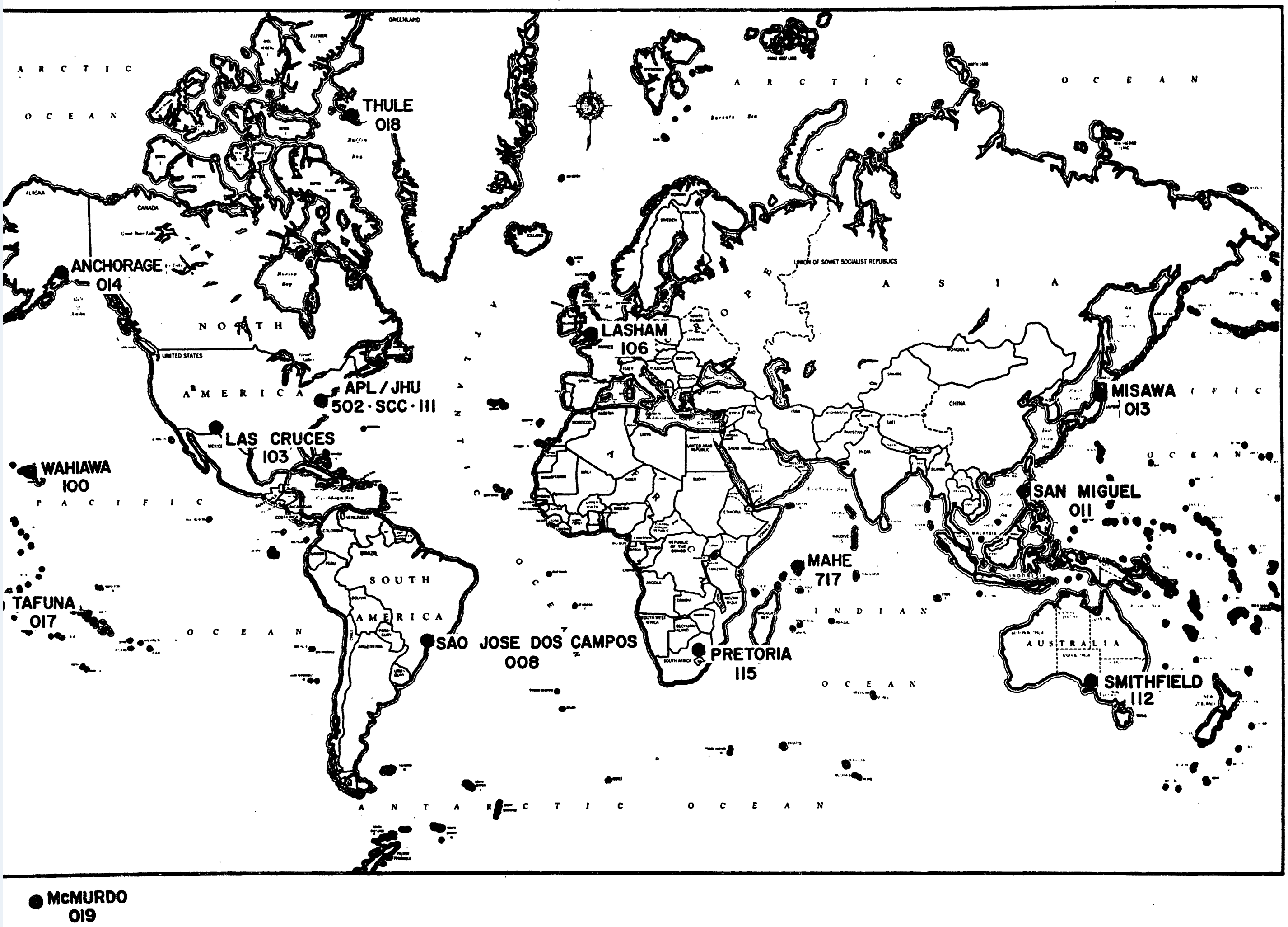

FIGURE 1. Locations and identifying numbers of the stations in the Tranet system.

perturbations becomes simpler and more reliable as we shorten the interval between observations. With the present system, any satellite having a minimum altitude of at least 900 km, is observed at least once each revolution and it is rare for the interval between observations to be as long as 1 h. The total observing time per day ranges from about 600 min for a satellite at low inclination to about 1000 min for a satellite in a polar orbit.†

† In the remarks about observations, no observations from a pass of the satellite above a station horizon are counted unless the satellite rises to at least 15° above the horizon. However, for a pass in which the satellite rises at least this high, individual observations are used for elevation angles down to about 7° in orbit determinations. Data are used down to the horizon for refraction research.

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The third criterion follows from the fact that orbital data from satellites in many different orbits must be analysed simultaneously. Since there are only a few active satellites at any one time, this implies using data obtained over many years. Further, since finding the coordinates of the stations from the satellite data is a necessary part of the research, it is desirable to keep the number of different sites as small as is consistent with gathering the necessary volume of data. Thus we prefer to use the same sites regardless of the satellite orbit, instead of, for example, moving stations to low latitudes for low inclination satellites and to high latitudes for high inclination satellites.

The operations of the communications centre and of the telemetry and command centre require great technical ability and are of great importance to the system. However, the details of their functioning are not needed in order to understand the uses of the data, and these components of the system will not be described here.

The stations make their observations in response to (*a*) 'alerts' and (*b*) a priority list. The alerts are prepared by the computing centre and give the predicted times of rise and set for each satellite along with other information to assist the station operators. The priority list is for use if two or more satellites are above the horizon at the same time, or if one follows another so closely that it is not possible to 'recycle' the station to make it ready for another set of observations. Broadly speaking, we give highest priority to a satellite that has just been launched, and maintain this priority for a few days. If there is no such satellite, we assign preference according to the uniqueness of the orbital parameters; that is, a satellite whose orbit differs considerably from that of any other satellite which has been observed is more likely to give new information and is accorded high priority. Among satellites whose orbits resemble each other, we give priority to the oldest, in order to improve the knowledge of long-period perturbations.†

Since one need of the geodesy programme is for data from a given satellite that are closely spaced in time, we do not 'split' the system. That is, we do not assign some stations to observe some satellites and other stations to observe other satellites, except occasionally for special purposes. If the needs for orbital information from different satellites conflict, we generally alternate the entire system between the satellites by means of the priority list, maintaining the concentration upon a given satellite over a period of several days. We can afford to do this because of the high accuracy that has been reached in orbital determination and in interpolating satellite position between determinations.

During a pass, which means the time of one passage of a satellite above the horizon, a station makes a measurement of the Doppler frequency every 4 s. Data from each pass are sent separately to the communications centre, using teletypewriter circuits. A pass record contains three segments.

The first segment is called the header. It gives the identifying numbers of satellite and station, the date, the rise time for the pass as listed in the alerts (*not* the observed rise time), the carrier frequencies for which the Doppler shift was measured, the best estimates of the current error in the station clock and frequency standard, and a quantity called N_c that will be discussed later.

The second segment contains the Doppler measurements and the times at which they

† These considerations reflect only the needs of the geodetic programme. The system supports other research and development programmes, whose needs are also considered in priority assignments.

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were made. The data are punched on to teletype tape by the station equipment, and this tape is fed into the teletypewriter for transmission to the communications centre. Several satellites also transmit timing signals. With these satellites, a station measures the time of reception of each signal against its local clock, and reports the timing data along with the Doppler data.

The third segment contains meteorological data that are used in calculating the refraction effect of the troposphere upon the measured frequencies. These data consist of the time at which the meteorological measurements were made, the barometric pressure, the relative humidity, the temperature, and code numbers to identify the nature and extent of cloud cover and of precipitation, if any. These measurements do not have to be made for each pass, and can be made only every few hours.

Except for Station 111, each station has a precision crystal oscillator and a clock driven by it to serve as the local frequency and time standards. In accordance with good practice, these standards are interfered with as little as possible. By means that will be described in the error analysis, each station monitors its local standards to obtain the clock and frequency errors in the header of each pass record.

The clock and frequency errors reported by the stations are used in initial processing of the data by the computing centre, but the final accuracy of the orbital and geodetic computations does not depend upon the accuracy of the reported values. Final accuracy depends upon the monitoring of all the local standards by Station 111, located at the Applied Physics Laboratory, and by the computing centre. This monitoring will be discussed in more detail later. It is made possible by two circumstances. First, Station 111 contains two independent caesium beam standards, which serve as working standards for the system. Secondly, assuming that the Station 111 standards are correct, the computing centre can compute the time and frequency errors for each pass from the Doppler and timing data, using values reported by the stations only to furnish the starting-point of the computation.

Accuracy of frequency at Station 111 is maintained by monitoring the caesium standards against each other and against the National Frequency Standard in Boulder, Colorado, using v.l.f. for the latter monitoring. Time accuracy is maintained by using data furnished by the Naval Observatory, which also monitors timing signals from those satellites that transmit them. Through these means, the National Frequency Standard becomes the primary frequency standard and the Naval Observatory becomes the primary time standard of the system. In addition, time accuracy is maintained by monitoring various radio services at both A.P.L. and the Observatory and comparing results.

The method of making Doppler and timing measurements has been described elsewhere (Weiffenbach 1960) and will only be summarized here. A received frequency is beat against a suitable frequency generated by the local standard. The local frequency is typically chosen to differ from the satellite transmitted frequency by 50 to 80 parts per million. The difference exceeds the maximum Doppler shift so that the beat frequency never passes through zero. The beat frequency signal passes through a tracking filter to reduce noise and then into a counter which counts a chosen number N_c of cycles of the beat frequency. Between the times that this count starts and stops, a second counter is counting cycles of a locally generated fixed frequency. The reading of the second counter is a measure of the time required for N_c cycles of the beat frequency, and it is this reading that is reported by the station as a 'Doppler measurement'. Some stations filter before instead of after beating with the local standard.

The Doppler shift is always measured for two or more carrier frequencies transmitted by the same satellite. Use of more than one frequency allows making a correction for refraction in the ionosphere. If the correction is to be accurate, the frequencies transmitted from the satellite must be controlled from the same oscillator. Further, all frequencies used in a station should be generated from a common oscillator (see §3).

The remaining sections of this paper will deal with the observational errors in the tracking system, that is, with the errors in the measured time and in the frequency calculated from the cycle counts. While these are the basic errors in the system, they have no obvious relation to errors in orbital determination or in geodetic work. Hence we shall also use another measure of error, based upon quantities evaluated at closest approach.

The instant $t_{\text{c.a.}}$ of closest approach can be calculated from the data. Let $\delta t_{\text{c.a.}}$ denote the error in the calculation of this instant. Further let $V_{\text{c.a.}}$ be the horizontal component of satellite velocity at this instant and let $E_A = V_{\text{c.a.}}\, \delta t_{\text{c.a.}}$. ($E_A$ will be called the along-track error.)

The range from the station to the satellite at $t_{\text{c.a.}}$ can also be inferred from the data. Let E_R, called the slant-range error, denote the error in the minimum range and let $E_T = (E_A^2+E_R^2)^{\frac{1}{2}}$. The quantity E_T will be the second measure of error to be quoted; E_A and E_R will also be given.

If station coordinates were known exactly, E_T would be approximately the error in the orbital position of the satellite at time $t_{\text{c.a.}}$. Conversely, if the satellite orbit were known exactly and if the data from a pass were used to locate the station position, E_T would be approximately the error in this position.

This is a good place to point out that only two coordinates, whether of a station or a satellite, can be inferred with high accuracy from a single pass. It is possible to calculate all six parameters of a satellite orbit (Guier & Weiffenbach 1958) from a single pass, but the position error even within the time span of the data from such a calculation is likely to be some hundreds of meters. To find accurately all three coordinates of a station, or to make an accurate orbital determination, it is necessary to use several passes.

For a particular pass, any type of frequency error can be written as the sum of three components: (*a*) the algebraic mean, (*b*) a component with zero mean that is symmetric about $t_{\text{c.a.}}$, and (*c*) a component that is anti-symmetric about $t_{\text{c.a.}}$. An error E_A results almost entirely from the symmetric component and an error E_R results almost entirely from the anti-symmetric one.

2. Tropospheric refraction errors

Refraction effects caused by the troposphere can be considered in terms of a refractive index n that is a function of position, with the propagation obeying Fermat's principle. Consequently, the portion of the optical path in the troposphere differs from the corresponding portion of the instantaneous slant range. The difference, called $\Delta s_{\text{tro.}}$, is not constant during a satellite pass, being obviously greater at low than at high elevation angles. The tropospheric contribution to the Doppler shift at any instant is

$$\Delta f_{\text{tro.}} = -\frac{f}{c}\frac{\mathrm{d}}{\mathrm{d}t}(\Delta s_{\text{tro.}}) \tag{2·1}$$

where f is the transmitted frequency and c the velocity of light.

An expression for $\Delta f_{\text{tro.}}$ can be derived (Hopfield 1963) on the basis of pass geometry and the following simplifying assumptions about the troposphere:

(1) The refractivity N of air (where $N = 10^6\,(n-1)$) is a continuous function of height above the Earth but is independent of horizontal position and of time, within the region and the time interval of a satellite pass. This assumption is violated near a weather front but is otherwise fairly accurate.

(2) The N profile (height variation of N) can be approximated by a quadratic expression that decreases from its value at the observing station to zero at a specified height above the geoid. This height is called the 'equivalent height' of the troposphere.

(3) Curvature of the signal path is negligible. For arrival angles close to the horizon, this assumption is seriously in error; consequently data are not used for low elevation angles of the satellite.

The resulting expression for $\Delta f_{\text{tro.}}$ was incorporated into the orbit computing program, starting in January 1964. A value of $\Delta f_{\text{tro.}}$ is computed as a correction for the observed Doppler shift at each data point of each pass, using the geometry of a preliminary orbit and the assumed refractivity profile. In the profile, the equivalent height of the troposphere is taken to be 23 km. Originally the surface refractivity was taken from a table (Bean & Horn 1958) which gives diurnal means for each season for each station. Since May 1964, we have had the necessary meteorological data reported by the stations, and we calculate the surface refractivity N_0 from the equation (Smith & Weintraub 1953)

$$N_0 = (77{\cdot}6/T)[P+(4810H/T)]$$

in which T (°K) is temperature, P (mb) is the surface pressure, and H (mb) is the product of the relative humidity by the saturation pressure of water vapour at temperature T.

In the absence of horizontal gradients of N, the tropospheric refraction contribution to the observed Doppler shift has the same sign as the total Doppler shift and is thus anti-symmetric. Hence, if the data distribution is reasonably uniform throughout a pass, tropospheric refraction when uncorrected produces only an error E_R in the minimum range. The size of E_R depends upon both the maximum elevation angle reached by the satellite during a pass and the lowest instantaneous elevation angle for which data are used. The refraction contribution of course also depends upon the surface refractivity.

The dependence of the refraction contribution upon the geometric factors is shown in figure 2. In this figure, the minimum range error (that is, what the error would be without a refraction correction) is plotted as a function of the maximum elevation angle, for several values of the data 'cut-off' angle, for the typical value $N_0 = 320$ and for a circular orbit of 900 km altitude. In orbital determination for geodetic purposes, we generally use a cut-off angle of 7° and do not use a pass for which the maximum elevation is below 15°. Thus in the worst case the tropospheric contribution reaches about 50 m.

Comparison of refraction contributions calculated with the assumed model and those calculated from observed profiles for many circumstances shows deviations of as much as 15%. However, with a maximum error of about 3%, the deviation (Hopfield 1965) is a linear function of N_0. Since about January 1965, the values of $\Delta f_{\text{tro.}}$ calculated from equation (2·1) and the assumed model have further been corrected by this linear function of N_0, which presumably arises from the difference between the quadratic profile and an accurate profile.

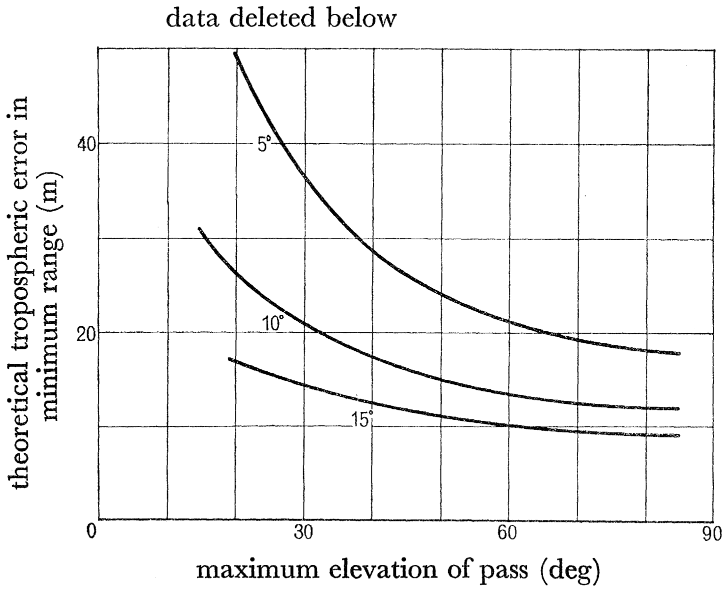

Figure 2. Contribution of tropospheric refraction to the calculated minimum range between a station and a satellite, as a function of the maximum elevation angle reached by the satellite during a pass and of the lowest elevation angle for which data are used.

The errors remaining after the computation of tropospheric refraction come from deviations between the instantaneous and average profiles, from deviations between instantaneous meteorological conditions and those measured as much as several hours before, from bending of the optical path which is neglected in the computation, and probably from other sources. The largest error is believed to be that coming from the age of the meteorological data; this can amount to as much as 10% of the tropospheric contribution. Thus the residual error can be as much as 5 m; the standard deviation of residual error is estimated to be 3 m, all in E_R.

Since the error is in the minimum slant range from the station to the satellite, errors in the latitude and longitude of the station tend to cancel when many passes are used. There is a tendency to bias the altitude of a satellite in an orbital calculation, but since the computed average altitude is dominated by the period and not by the ranges, the bias is much smaller than the average range error.

3. Ionospheric refraction errors

Ionospheric and tropospheric refraction differ in a microscopic sense but have qualitative similarities in a macroscopic sense. The ionospheric contribution $\Delta f_{\text{ion.}}$ to the measured frequency, can be calculated from an equation like (2·1) with appropriate methods for estimating $\Delta s_{\text{ion.}}$ from a model of the ionosphere. $\Delta f_{\text{ion.}}$ depends upon the transmitted frequency f and can be expanded in a power series in f (Guier & Weiffenbach 1960):

$$f_{\text{ion.}} = -(f\dot{\rho}/c) + \sum_{1}^{\infty} (a_i/f^i), \tag{3·1}$$

in which ρ is the instantaneous range to the satellite and $\dot{\rho}$ is its time derivative. The coefficients a_i are independent of frequency. They depend explicitly upon time because ionospheric densities vary with time, and they depend implicitly upon time because they depend upon the positions of the satellite and of the observer.

In the Tranet system, Doppler frequencies are always measured simultaneously on at least two carrier frequencies. When two frequencies are used, the coefficients a_i with $i \geqslant 2$ in (3·1) are set equal to zero and the values of $\dot{\rho}$ and a_1 are calculated for each measurement. Data using more than two frequencies have been used for ionospheric research; only 'two-frequency' data have been used in orbit determination. The error remaining because the higher order a_i are not zero will be called the 'residual ionospheric error'. It should be pointed out that the value of a_1 calculated from two frequencies is not in general the correct value.

It is important that the frequencies transmitted from the satellite be controlled by the same basic oscillator. It is also important that the frequencies used for reference at a station also be derived from a common oscillator. The reason for this is to avoid degrading the accuracy because of phase jitter in the oscillators, when solving the simultaneous measurements for $\dot{\rho}$ and a_1.

The residual ionospheric error in the absence of horizontal gradients in the ionosphere is anti-symmetric. The corresponding position error is in the minimum range E_R with no contribution to E_A, just as with tropospheric refraction.

Measurements made at the University of Texas (Willman & Doyle 1963) indicate that the residual ionospheric error frequently has a symmetrical component that may be as large as the anti-symmetrical one. Most of the data on the residual error come from there. Figure 3 (reproduced from the reference by permission) shows an example in which the residual error is extraordinarily large. The ordinate in figure 3 is the estimated value of the term a_3/f^3 evaluated for $f = 54$ Mhz, point by point throughout a particular pass.† The figure also gives geometric and time information about the pass. The satellite number given is the digital equivalent of the international number 1961 o1. Station 92 is located at Austin, Texas, and is not in the present system. The total residual refraction contribution to the position error for the pass shown in figure 3 is about 20 m.

The standard deviation of the residual ionospheric error for the data obtained in Texas, if the geometric conditions of at least 15° maximum elevation and 7° cut-off elevation are used, is 2 m. In spite of this small value, the residual ionospheric error is of considerable concern for the following reasons:

(*a*) The error may be larger at other sites.

† The term $a_3 f^3$ was evaluated on the assumption that $a_2 = 0$. Calculations of a_2 and a_3 made with idealized models indicate that, at the frequencies currently used for Doppler observations (54 to 400 Mhz), the term in a_3 dominates the term in a_2 whenever either is large enough to produce appreciable error. If a_2/f^2 had been evaluated from the data on the assumption $a_3 = 0$, the size of the computed errors would have been different.

(*b*) The data were obtained during relatively low solar activity. Theoretical estimates of the error during maximum solar activity give values as much as two orders of magnitude higher.

(*c*) The residual error, averaged over all passes, does not tend to cancel completely but instead tends to give a bias in the latitude of the observer.

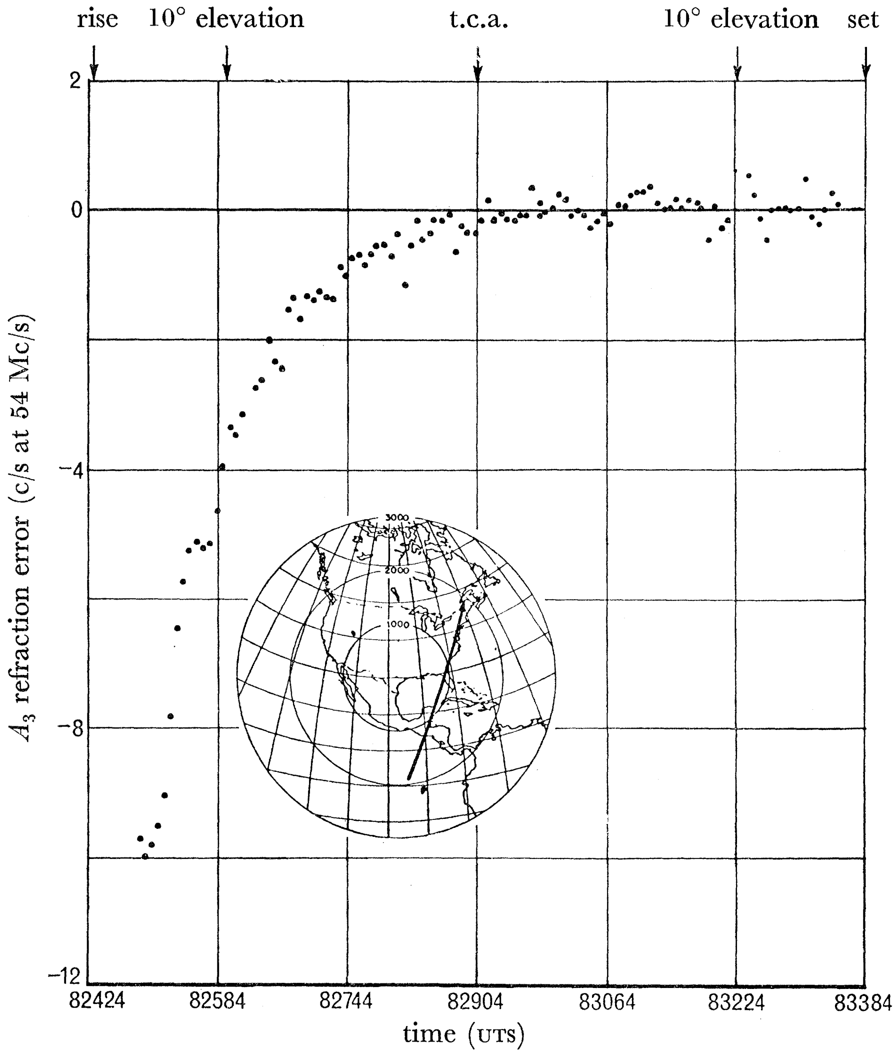

FIGURE 3. The uncorrected part of the contribution of ionospheric refraction to the Doppler frequency as a function of time during a pass, for a pass in which this contribution is extraordinarily large. Station 92; day 111 year 1962, Sat. 61151; max. el. 28·79, t.c.a. 82913; az. (t.c.a.) 106, S → N.

Position errors (m) for 150–400 pair:

cutoff	lat.	long.	C	L	t.e.
0°	−16·0	12·3	16·2	−12·1	20·2
10°	− 6·8	5·9	7·6	− 4·9	9·1

For the present, we shall use 2 m as the residual ionospheric error, but with the caution that it may be much larger at some times and places. We shall use 1·4 m as the contribution to each of E_A and E_R.

4. Timing errors

Time enters into the operations and computations of the Tranet system in several distinct ways:

(*a*) Time is used to relate the position and motion of the stations to the celestial sphere. The time needed here is UT 1.

(*b*) Time is used to compute the positions of the Sun and Moon, the position of the equinox, and the obliquity of the ecliptic. The time needed here is Ephemeris Time (ET).

(*c*) Time is used as the independent variable in the equations of motion of the satellite. The same time base should be used to furnish the time portion of a Doppler frequency *vs.* time measurement. Ideally this time base should probably be ET. In day-to-day operations, any time base with a linear relation to ET will serve.

The accuracy needed in these three types of time varies considerably.

The quantities in item *b* for which ET is needed vary slowly and large time errors can be tolerated. At present we can ignore the difference, about 35 s, between ET and UT.

If the values assigned to UT 1 for a given orbit computation are in error only by a constant, the only effect is to give a wrong orientation of the orbit with respect to the celestial sphere. In some studies, such as of the secular and long-period behaviour of the node, an error in UT 1 is important. In most studies to date, such as evaluating non-zonal harmonics of the gravity field, an error in the values assigned to UT 1 has no consequence provided that the correct rotation rate of the Earth is used.

The most strict demands for timing accuracy come from item (*c*) above. An error δt in time here makes a position error $v\delta t$ where v is the satellite speed; for estimating errors we take v to be 7 m/ms. Clearly neither epoch nor rate errors in this time with respect to an external standard such as UT or ET matter. Only lack of synchronization among the stations is significant. In spite of this fact, it is operationally convenient to have the stations adhere as closely as possible to an external standard.

The external standard chosen is UT 2*c*, the time base provided by the coordinated radio transmissions, even though this time does not meet any of the requirements stated above. It is convenient because a station can readily establish synchronization within a few milliseconds in the event of a malfunction. It flows with a linear relation to ET to high accuracy, except for the occasional discontinuities that are introduced deliberately. It costs little in a geodetic programme to avoid orbital determination over an interval that includes a discontinuity. For long-term studies such as those of the nodal precession, we can readily correct for the difference between UT 2*c* and UT 1.

The exact procedure for establishing and maintaining synchronization varies somewhat, but the following is typical. A station initially establishes synchronization by reception of timing signals such as those from WWV, using the best estimate of propagation delay. Experience has shown that the error at this stage can be as much as about 3 ms, depending upon location. Simultaneously the station begins phase comparison between its local frequency standard and a v.l.f. transmission. Averaged over a day, this allows maintaining synchronization within the accuracy allowed by the quality of the local standard. After the computing centre has analysed the satellite timing data from the station for a few passes, the station re-establishes synchronization as accurately as the satellite data allow. Synchronization is also checked occasionally by carrying an accurate portable clock between stations.

Table 1 shows the quality of time synchronization maintained at the stations. Since the stations monitor their clocks and do not necessarily reset them, the clock errors in table 1 are the differences between the clock errors as estimated locally and as estimated by analysis of satellite timing data; they are not the errors in the actual clock readings. The numbers in table 1 are for the period January to March 1966.

Table 1. Clock errors at Tranet stations over a sample 3-month period

station number	mean error (μs)	S.D. (μs)
111	0†	21‡
008	292	524
011	79	217
013	−80	486
014	−85	479
017	163	429
018	32	142
019	−95	477
100	47	195
103	55	413
106	−174	275
112	−28	158
115	50	257
717	−74	276
system	14	358

† The clock errors in this table were computed with reference to the Station 111 clock as the working standard and not with reference to the primary standard of the Naval Observatory. Hence the mean error for Station 111 is zero by definition.

‡ This is the standard deviation of the error in recovering satellite timing signals, on a 'per pass' basis, and does not refer to error in the Station 111 clock.

Before we discuss table 1, a remark should be made about the distribution of errors. The distribution for a particular station is approximately normal up to about 1·5 ms. There are few errors between about 1·5 and 3 ms, although the number seems to be greater than expected from the distribution of smaller errors. The number of errors exceeding 3 ms, though small (about 1%), is enormously greater than we expect from a normal distribution. We conclude that the errors exceeding 3 ms do not belong to the same populations as the smaller errors. Since we can readily detect these errors and since we have enough data that we can afford to omit any data containing them, we have omitted them from consideration in table 1. They may result from malfunctions that have gone undetected for a short time.

Table 1 shows the algebraic mean of the clock errors and the standard deviation of the clock errors from zero for each station. The table also shows the mean and the standard deviation for the entire system, excluding Station 111, which is a special case. The mean error for the system is 14 μs and the standard deviation is 358 μs.

The clock errors in table 1 are taken with reference to Station 111, which is the working standard of the system. Hence the mean clock error at Station 111 is zero by definition. The error of Station 111 with respect to the primary standard is rarely more than a few microseconds and is neglected. The standard deviation entered for Station 111 in table 1 does not refer to the clock at Station 111. It is rather the estimated precision of recovering time from the satellite timing signals. This estimate is obtained as follows:

The satellites that transmit timing signals emit a signal every 2 min. The times of reception of these signals are recorded at Station 111 with respect to its clock and are first corrected for the light time from satellite to station. For a given day, the best linear fit of the corrected times to the Station 111 clock is found, and the standard deviation of the individual time points from the linear fit, which is taken as the calibration of the satellite clock for the day, is calculated. On the average, 28^+ timing points are received from a particular satellite per day and 7^+ are received per pass. The entry of 21 μs in table 1 is the standard deviation of the individual residuals divided by $\sqrt{7}$. Thus it is the estimated accuracy of recovering time from the satellite signals provided that the clock calibration is used in the recovery. It is believed to be the accuracy and not merely the precision, because the jitter in the satellite clocks is believed to be small compared with 21 μs.

Using data obtained in the last quarter of 1963, Markowitz (1964) estimated 33 rather than 21 μs. The difference probably results from improvement in techniques in the intervening two years.

The main source of the error in time recovery is probably the variability of the signal delay in the stations. The signal delay is calibrated frequently. It changes because of drifting in equipment characteristics, and it also varies with the strength of the received signals. It is rather difficult to lower the variability due to signal strength, at least until stronger signals are available. With present signals, meticulous alignment and calibration are needed to keep the variability below 25 μs.

Reverting to table 1, the standard deviation of 358 μs corresponds to an along-track error of about $7 \times 0{\cdot}358 = 2{\cdot}5$ m. In present practice, this is the error attributable to timing errors. Because it is not an important error at present, we have not actually gone to the process of correcting times according to satellite timing data on a point-by-point basis. Correcting the reported times would require additional computing, and the small improvement in accuracy does not so far warrant the added cost. Presumably the error in existing data could be reduced to the level of variability, about 21 μs if Station 111 time recovery gear is typical, corresponding to about 0·15 m. Station 111 is probably not typical, since we have not required other stations to be as meticulous in their timing operations. The irreducible timing errors in existing data are probably about 100 μs.

In summary, we shall take 2·5 m as the error contribution from clock errors with present programmes. This error is along-track. It can be reduced to less than 1 m by post-analysis of satellite timing data.

There is another source of timing error that comes not from the clocks but from the measurement method. The count of the beat frequency, referred to in §1, begins on the first positive-going zero crossing of the beat signal that occurs after an integral second, but the digital circuitry records only the integral part of the second and not the fraction of a second that elapses before the count begins. On the average, a count begins one-half cycle of the beat frequency after the second, and this value is added to the time recorded in the computing program. This leaves a random time error whose maximum value is half the period of the beat frequency. The size of this error depends upon the beat frequency chosen for use and varies from station to station. At three of the stations, this error has a maximum value of 54 μs and a standard deviation of 31 μs. At the other stations, these numbers are smaller by a factor of 3.

5. Frequency errors

It is useful to begin the discussion of frequency error sources with the mention of a non-source. A smooth drift in the frequency of the satellite-borne oscillator does not produce any error. Suppose, for example, that the satellite frequency is a linear function of time containing two parameters. These parameters are estimated at the same time as the orbit parameters and hence the fact that they are not known *a priori* does not cause any error. Put another way, the motion of the satellite does not interfere with the ability to measure the oscillator frequency, it merely makes the measurement more difficult. It is not necessary that the frequency variation be linear. Any reasonably smooth variation can be computed. With good oscillators at controlled temperatures, a linear variation is sufficiently accurate for present use.

There is an interaction between frequency drift and frequency errors. The mere fact that the frequency parameters must be calculated increases the sensitivity of calculated positions to frequency error sources. This added sensitivity is probably of the order of 10%, but it has not been studied thoroughly.

Since it is assumed that the station oscillators have constant frequency, their drift does constitute an error source. The drift of the oscillators used in the stations varies from 20 to 200 parts in 10^{12} per day, as measured at Station 111 before the oscillators were installed. The worst of these rates causes a contribution to the minimum range error E_R of a few centimetres.

We now turn our attention to sources of frequency error that we shall call noise. By noise, we mean frequency errors that are not correlated for more than a few seconds.

(*a*) *Oscillator noise.* A type of stability curve of an oscillator can be obtained by measuring its average frequency over successive equal time intervals, forming the differences between successive values of the average, and calculating the standard deviation of the differences. The time interval used will be called the averaging time, the standard deviation will be called the stability, and the stability is a function of the averaging time. Stability curves have been measured for the oscillators used in all the stations and the satellites.

Figure 4 shows the extremes of the stability curves of the station crystal oscillators plotted on a logarithmic scale. For short averaging times, the stability is governed by random phase jitter, and the stability curve follows a straight line of slope $-\frac{1}{2}$. For long averaging times, the stability curve is governed by the ageing rate of the crystals, and follows a straight line of slope 1. The effect of the ageing rate component, which ranges from 20 to 200 parts in 10^{12} per day, has already been discussed. Here we are concerned with the random component, which ranges between 30 and 70 parts in 10^{12} for 1 s averaging. The random jitter and ageing rate components of the satellite oscillators lie in the same ranges as those of the station oscillators.

Guier & Weiffenbach (1960) give the following semi-empirical formula for the position error resulting from noise:

$$E_T = (2{\cdot}5 \times 10^{11}/\sqrt{N})(\sigma_f/f) \text{ metres,} \tag{5·1}$$

in which N is the number of data points for which the standard deviation of the frequency noise error is σ_f. Using 60×10^{-12} as the stability of the oscillators, both station and satellite, for 1 s averaging, we divide this number by $\sqrt{2}$ to get the noise, since the number came from differences between two readings. We then multiply by $\sqrt{2}$ since two independent oscillators are involved, leaving 60×10^{-12} as the resultant noise including both oscillators. A station obtains about 150 data points for a typical pass. Each point represents 0·5 s averaging time, so there are the equivalent of 75 data points with 1 s averaging. Equation (5·1) then gives 1·7 m as the total error contribution from oscillator noise.

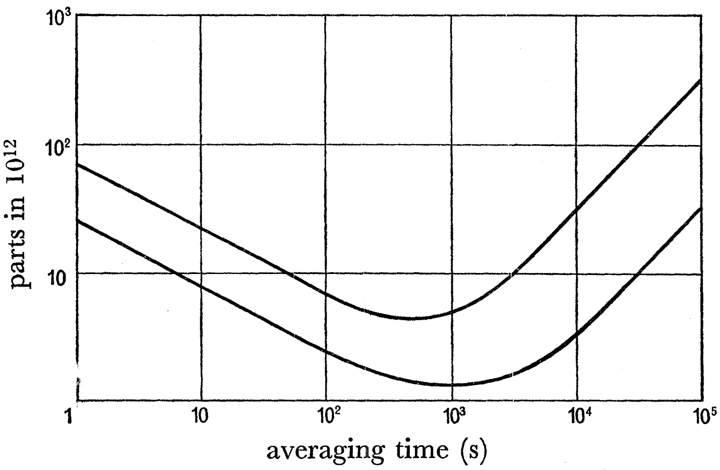

FIGURE 4. Extremes of the stability curves of the oscillators used in the observation stations.

(*b*) *Measurements of total system noise*. The 'data editor' described by Guier (this volume, p. 89) yields the best estimate of the point-to-point noise in the data along with other indications of data quality. This estimate applies to the total system noise arising from all sources.

Table 2 shows the noise levels and radiated power for several satellites. Instead of radiated power, it would have been better to give the r.m.s. value of signal strength taken over all pass geometries, but this number is hard to come by. We conclude from table 2 that the total noise level has not changed significantly since 1962. We see that the noise decreases slowly as the radiated power increases, and we tentatively conclude that most of the noise is not ambient noise entering the antenna but is rather noise generated within the equipment. The oscillator noise does not contribute significantly to the total.

The r.m.s. of the noise levels listed in table 2 is 260 parts in 10^{12}. For this value and $N = 150$, the corresponding position error is 5·3 m; for different satellites this error ranges from 3·0 to 7·1 m. The position error is divided evenly between E_A and E_R.

We can only speculate where most of the noise comes from. It apparently does not come from the noise figure of the receivers used. It may be jitter or hunting in the many phase-locked loops involved in the total equipment. The noise coming from truncation or rounding of digital information, whether in the station equipment or in computing processes, is less than 100 parts in 10^{12}.

Since the total frequency noise is measured to be 260 parts in 10^{12}, and since we attributed 60 parts in 10^{12} to oscillator noise, we attribute $[(260)^2-(60)^2]^{\frac{1}{2}} \approx 253$ parts in 10^{12} to 'other measured instrumentation noise'. The corresponding contribution to E_T is 5·0 m.

TABLE 2. SATELLITE POWER LEVELS AND TOTAL SYSTEM NOISE

satellite	radiated power (W)†	total system noise (parts in 10^{12})
1962 $\beta\mu$ 1	0·25	280
1963 49B	1·0	170
1965 32A	0·1	350
1965 48A	1·0	150
1965 89A	0·25	230

† For the transmitter with the lower power level.

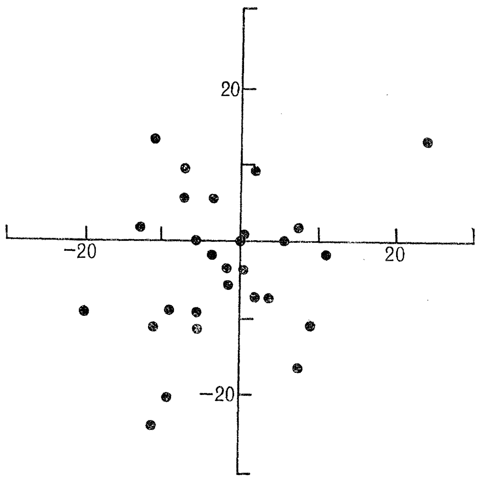

FIGURE 5. Computed positions of one station with respect to a second nearby station, after subtracting their known separation. Each point was calculated from the data for a single pass. The scales are in metres. (Courtesy North-Holland Publishing Co.)

The stations take a frequency sample with an averaging time of about 0·5 s every 4 s. If the observed frequency errors are as random as they seem to be, the noise could be cut by a factor of $\sqrt{8}$ by going to continuous rather than intermittent sampling.

(*c*) *Measurements of the random component of position error.* On several occasions we have located two stations within a few metres of each other and operated them independently, in order to assess their performance. For a particular pass, let ΔE_A and ΔE_R be the differences in the values of E_A and E_R found for the two stations from the Doppler data, after subtracting the appropriate components of the vector joining their two locations.

Many sources of error cancel completely in forming ΔE_A and ΔE_R. Among these are errors in satellite position and velocity, refraction errors whether systematic or random, and variations in the frequency transmitted from the satellites. None of the cancelling effects make a significant contribution to random errors in the system, so that ΔE_A and ΔE_R should furnish an excellent measure of the precision of the observing system.

Figure 5 shows the results of a typical test of this type, using data obtained during June of 1965 (Newton 1967). Each point in figure 5 gives the values of ΔE_A and ΔE_R for a particular pass. The distribution of the points is essentially circular. The standard deviation of $\Delta E_T = [(\Delta E_A)^2 + (\Delta E_R)^2]^{\frac{1}{2}}$ is 13·3 m. Since this is the result of two independent measurements, the standard deviation of a single measurement should be 9·3 m.

The error sources previously discussed do not account for this much error. The timing contribution is negligible. Although the clocks were independent, they were in an excellent position for maintaining synchronization, and probably deviated less than 100 μs during the tests. The total measured noise level accounts for about 5·3 m. This leaves an error contribution of amount $[(9\cdot3)^2-(5\cdot3)^2]^{\frac{1}{2}} = 7\cdot6$ m to be accounted for.

The origin of this error, which is currently the largest measurement error in the system, has yet to be found. It does not contribute to the second-to-second noise in the frequency measurements. It is random from one pass to another. Hence it is an error that remains correlated for at least the length of a pass, about 1000 s, but which is decorrelated between passes, an interval of about 6000 s. It affects E_A and E_R about equally, and contributes 5·4 m to each.

6. Summary and discussion

Table 3 summarizes the contributions to the observational errors of the Tranet Doppler System, as they exist at the present time. Some contributions vary slightly with the satellite being tracked; for these the standard deviation over several recent satellites is used. Only sources that contribute as much as 1 m are included. The resultant of all known sources is given separately; this is then augmented by the unknown source of §5(*c*), which has a correlation time of the order of an hour.

Table 3. Summary of error contributions

	error contribution (m)		
source	E_A	E_R	E_T
residual tropospheric refraction	0	3·0	3·0
residual ionospheric refraction	1·4	1·4	2·0
timing	2·5	0	2·5
oscillator noise, total for both station and satellite oscillators	1·2	1·2	1·7
other measured instrumentation noise	3·5	3·5	5·0
resultant error from all known sources	4·7	5·0	6·8
unknown source with about 1 h correlation time	5·4	5·4	7·6
total system error	7·2	7·4	10·2

The error analysis can be expected to vary with circumstances. The only circumstance we can think of that is likely to increase the total error is an increase in ionospheric activity. The mechanisms that cause the residual ionospheric error are not well known at present. By some estimates, this error may be as much as 200 m at some times and places if no corrective action is taken. Possible corrective actions include the use of higher frequencies, the use of more than two frequencies, and gaining an improved knowledge of the mechanisms of the error. We can expect to learn more about the mechanisms as the error becomes large enough to study during the next few years of expected increase in ionospheric activity. With corrective action, it is doubtful that the error will ever in fact be close to 200 m.

On the side of decreasing errors, we can cut the error due to timing, using existing data, to perhaps 0·7 m by an elaboration of the orbital computation programs. We should be able to cut the effects of frequency noise by a factor of $\sqrt{8}$ by taking continuous frequency samples rather than samples of 0·5 s each 4 s. The most obvious need is to study the unknown source.

Most of the results reported in this paper came from the efforts of many people working together, so that it is impossible to identify their individual contributions. Where possible I have identified specific contributions by references to the literature. In addition, I wish to thank the following people, who are all on the staff of this Laboratory, for the following: Mrs H. S. Hopfield for the computations embodied in figure 2, Dr G. C. Weiffenbach for the data used in figure 4, and Mr C. A. Dunnell for preparing figure 5 from computed values of E_A and E_R. Computation of these values, and of the clock errors in table 1, was carried out at the U.S. Naval Weapons Laboratory.

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